# Refinement of the Acoustoelasticity Theory in Transversely Isotropic and Quasi-Incompressible Media

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## INTRODUCTION

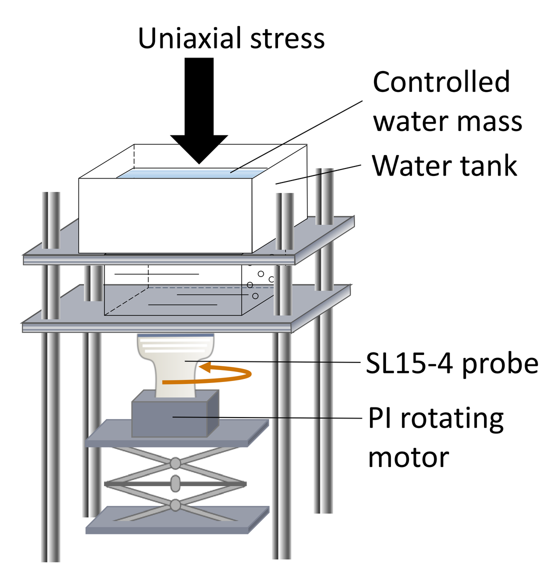
Nonlinear (NL) shear elasticity quantification has become a new complementary measurement to that of shear modulus characterizing soft tissues linear elasticity. It relies on acoustoelasticity (AE) theory. First developed for isotropic soft tissues, this technique consists in deducing the NL shear modulus from the evolution of the shear wave speed in uniaxially stressed media[1]. The implementation of AE in transverse isotropic (TI) soft tissues such as muscles requires refinements to include the specificities of the TI symmetry. The adaptation of the AE theory in TI quasi-incompressible media was previously developed[2], but only covers the 9 simplest configurations where the principal direction of the TI medium, stress, polarization $\vec{u}$ and propagation $\vec{k}$ directions of the shear waves are either parallel or perpendicular to one another. As a result, the first *ex vivo* estimations of muscle NL elasticity are highly biased since it is really hard to experimentally match the 9 configurations previously defined[2]. Therefore, the goal of our work is to consider the angle dependency of stress, $\vec{u}$ and $\vec{k}$ directions with respect to the principal axis.

## METHODS

→ **Measuring SH shear wave speed in TI media under known uniaxial stress.**

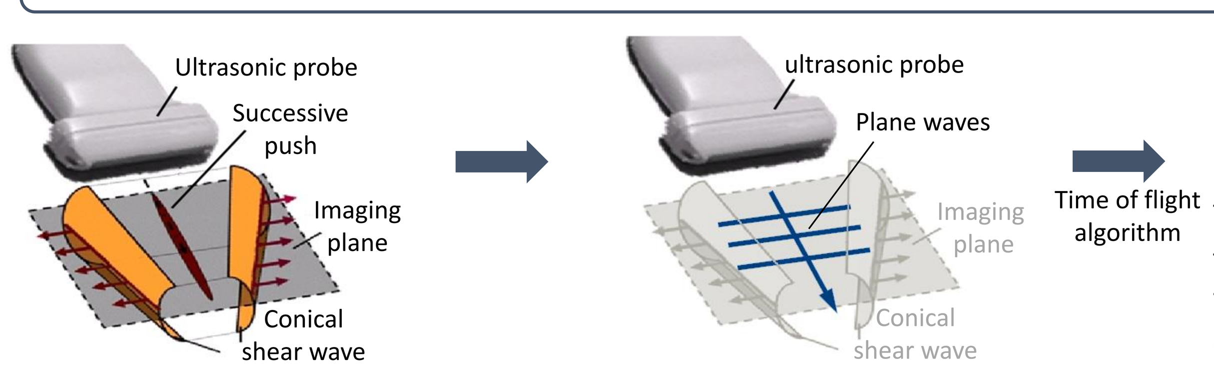
### Experimental setup for angle resolved AE experiments

V12 Aixplorer, Supersonic Imagine



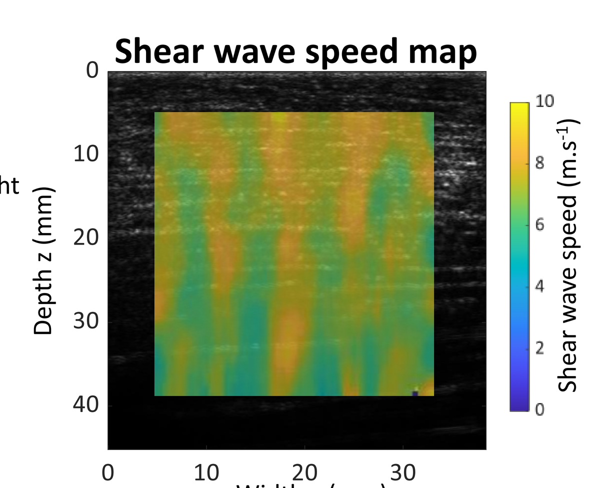

A SL15-4 probe (9 MHz central frequency, 256 elements) was rotated below the sample by using a motor, to carry out AE measurements considering SH mode waves propagating in any direction relative to the principle axis.

### Supersonic Shear Imaging technique



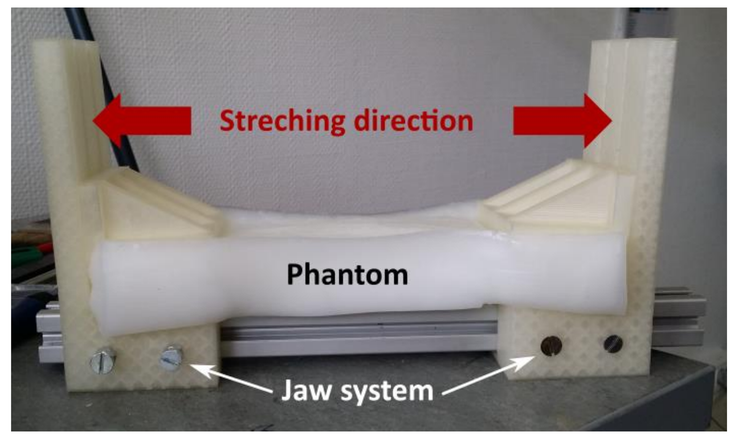

**Generation of a supersonic shear wave** using the ultrasonic radiation force

**Ultrafast imaging** of the shear wave propagation (8000 frames.s$^{-1}$).




### Studied TI medium: TI PVA phantom[5]

10% PVA – 1% Sigmacell type 20
2 isotropic freezing-thawing cycles
3 anisotropic freezing-thawing cycles (phantom stretching)



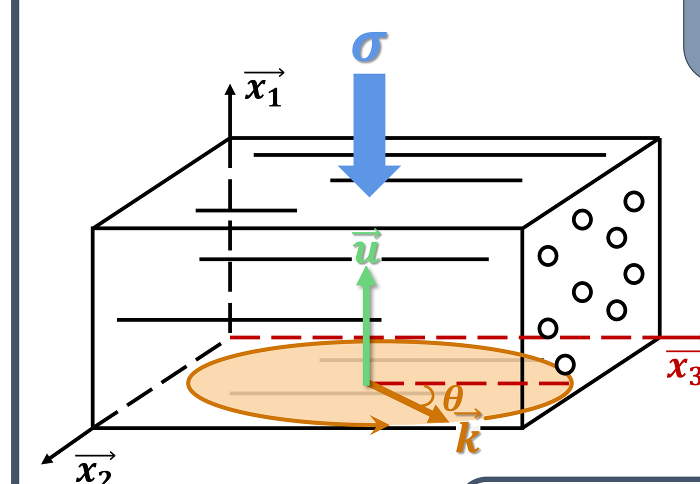

## THEORY

→ **Expressing the speed of SH (Shear Horizontal) mode elastic shear waves in an uniaxially stressed (along $\overrightarrow{x_1}$) TI quasi-incompressible solid.**

Strain energy[3,4] $e$:

$$e = \mu_\perp I_2 + \left(\frac{E_\parallel}{2} - \frac{3\mu_\perp}{2}\right)\varepsilon_{33}^2 + 2(\mu_\parallel - \mu_\perp)(\varepsilon_{13}^2 + \varepsilon_{23}^2) + \frac{A}{3} I_3 + (G + H + J)\varepsilon_{33}^3 + (H + 2J)\varepsilon_{33}(\varepsilon_{13}^2 + \varepsilon_{23}^2) + J\varepsilon_{33}(\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2)$$

(with $\varepsilon$ the Green-Lagrange strain tensor, $I_2 = \mathrm{Tr}\,\varepsilon^2$, $I_3 = \mathrm{Tr}\,\varepsilon^3$)




1st Piola-Kirchhoff stress tensor: $P_{ik} = \dfrac{\partial e}{\partial\left(\dfrac{\partial u_i}{\partial a_k}\right)}$

→ Particle motion equation: $\rho_0 \ddot{u}_i = \dfrac{\partial P_{ik}}{\partial a_k}$ (with $\vec{u} = \overrightarrow{u^D} + \overrightarrow{u^S}$)

→ Wave equation → Hooke's law for static strain expression

**Hypotheses for SH mode plane waves (→*implying* $\overrightarrow{u^D}$):**

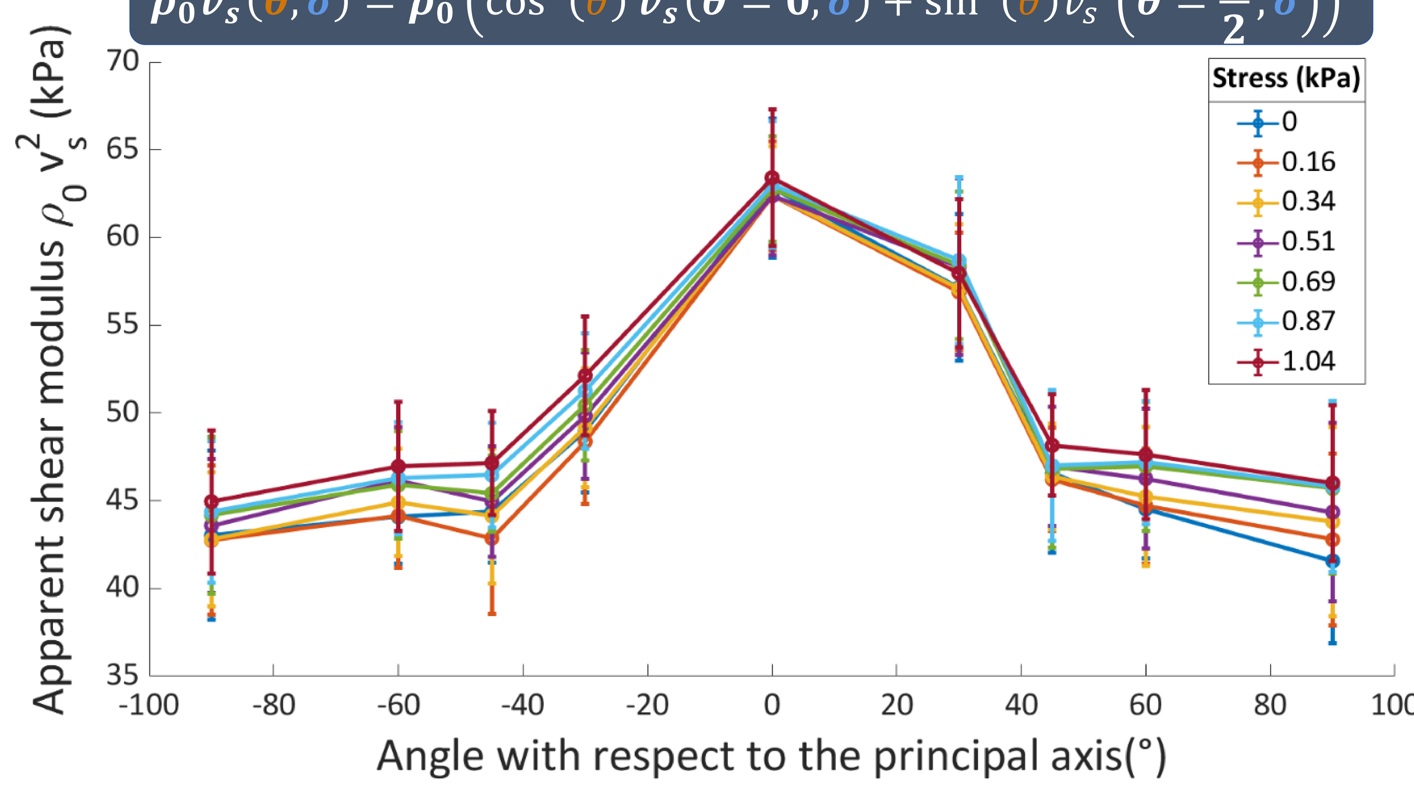
- $\overrightarrow{u^D} = u_1^D \overrightarrow{x_1}$
- $u_1^D = f(\vec{k}.\vec{r} \pm v_s t)$ with $\vec{k} = \begin{pmatrix} 0 \\ \sin\theta \\ \cos\theta \end{pmatrix}$

$$\rho_0 v_s^2(\theta,\sigma) = \mu_\parallel \cos^2(\theta) + \mu_\perp \sin^2(\theta) + \left[\sin^2(\theta)\left(E_\parallel - 2\mu_\perp - \frac{A}{2} + J\right) + \cos^2(\theta)\left(\mu_\parallel - \mu_\perp + E_\parallel - \mu_\parallel\frac{E_\parallel}{\mu_\perp} + \frac{A}{4}\left(1 - \frac{E_\parallel}{\mu_\perp}\right) + \frac{H + 2J}{2}\right)\right]\frac{\sigma}{2E_\parallel}$$

Where $\vec{u}$, $\overrightarrow{u^S}$, $\overrightarrow{u^D}$ are the total, static (due to static stress), and dynamic (due to the shear wave) displacement vectors, respectively. $\vec{a}$ the position in Lagrangian coordinates, $\vec{k}$ the propagation vector, $\theta$ the angle between the principal axis and $\vec{k}$, ($\mu_{//}$, $\mu_\perp$, $E_{//}$) the linear elastic coefficients, ($A$, $G$, $H$, $J$) the third order elastic coefficients, ($I_2$, $I_3$) invariant of the strain tensor and $\rho_0$ the density.

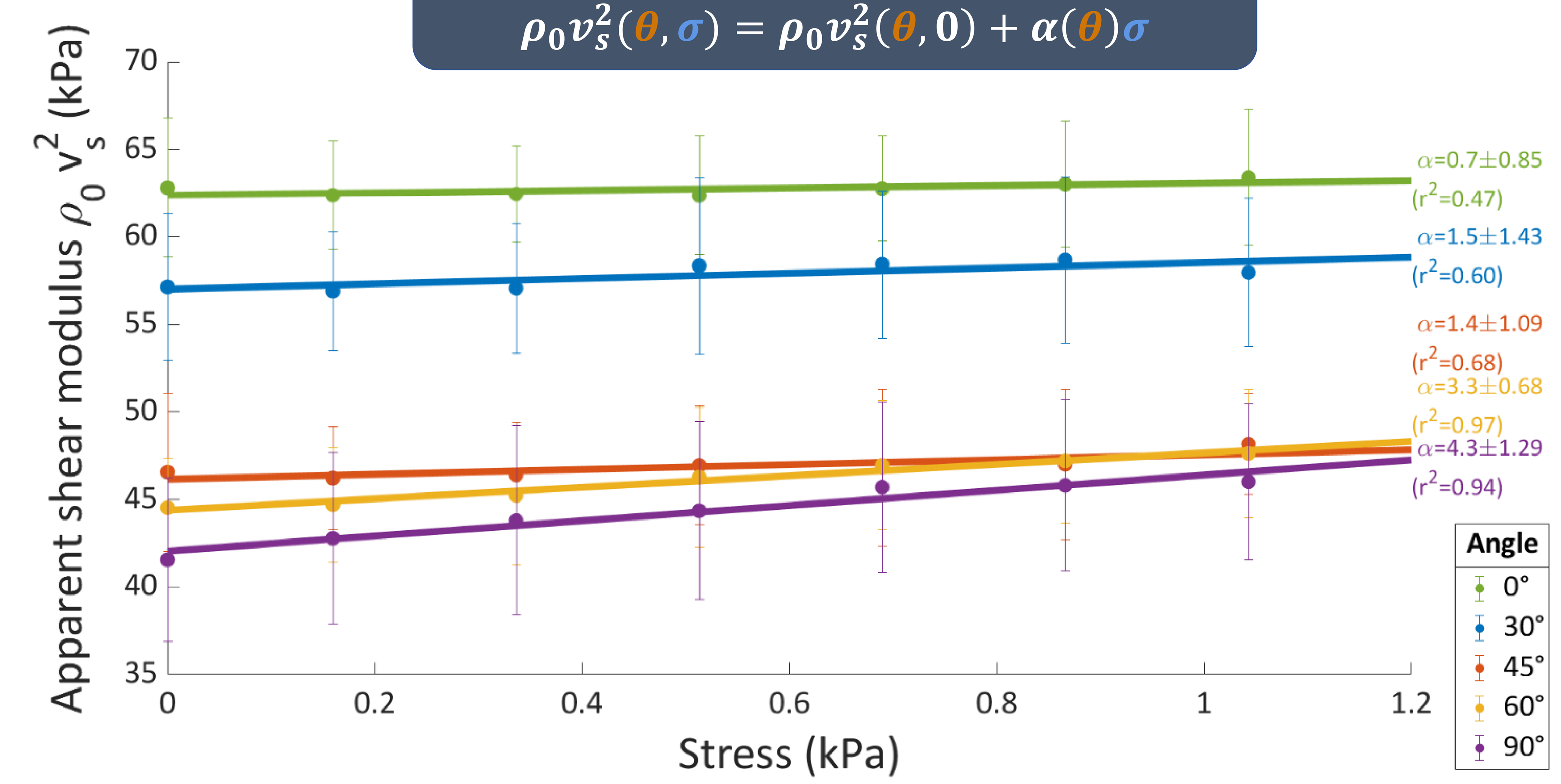
## RESULTS

$$\rho_0 v_s^2(\theta,\sigma) = \rho_0\left(\cos^2(\theta)\, v_s^2(\theta = 0,\sigma) + \sin^2(\theta)\, v_s^2\left(\theta = \frac{\pi}{2},\sigma\right)\right)$$




**Linear $\rho_0 v_s^2$ - $\sigma$ relationship**

$$\rho_0 v_s^2(\theta,\sigma) = \rho_0 v_s^2(\theta, 0) + \alpha(\theta)\sigma$$




## DISCUSSION

- The angle resolved AE equation for SH mode shear waves in TI medium was developed and verified experimentally on a TI PVA phantom.
- Since our development is based on SH mode shear waves, any experimental application of our equation requires (1) to identify the precise direction of the principal axis of the medium and (2) to place the probe parallel to a plane containing the principal axis, which is challenging on *ex vivo* muscle tissues, as well as on *in vivo* pennate muscles. Further development considering SV mode shear waves would complete our work.
- The retrieval of the complete set of muscles' NL parameters requires the use of other complementary AE configurations (rotation of stress, polarization and propagation directions), as well as an independent estimation of $E_\parallel$.

1/ Gennisson J.-L. *et al.*, JASA 2008; 2/ Bied M. *et al.*, IEEE IUS 2020; 3/ Johnson G.C., J. Nondestruct. Eval. 1982; 4/ Destrade M. *et al.*, JASA 2010; 5/ Chatelin S. *et al.*, PMB 2014.