Acoustoelasticity in transverse isotropic soft tissues: Quantification of muscles' nonlinear elasticity

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INTRODUCTION

Quantification of the elastic nonlinearity of biological tissues is of increasing interest in the early diagnosis of pathologies, such as breast lesions[1]. Measurement of nonlinear shear modulus (NLSM) in biological tissues using shear wave elastography relies on acoustoelasticity (AE). It consists in measuring the shear wave velocity v_s under uniaxial stress. The AE theory previously developed in isotropic quasi-incompressible materials[2] is revised in transversely isotropic (TI) medium to investigate muscle non linearity. In isotropic media, 3 configurations differing in the relative orientation of the uniaxial stress with respect to the polarization and the propagation directions of the shear wave have been identified, and the corresponding relationship between v_s, the local stress (σ), the linear shear modulus (μ) and the NLSM (A) has been derived for each case. In TI medium, such as muscle, this approach is no more valid due to the axis of symmetry. The goal of this work is to transpose the AE theory to TI soft tissues for a better understanding of the mechanical behaviour of muscles and pathologies.

METHODS

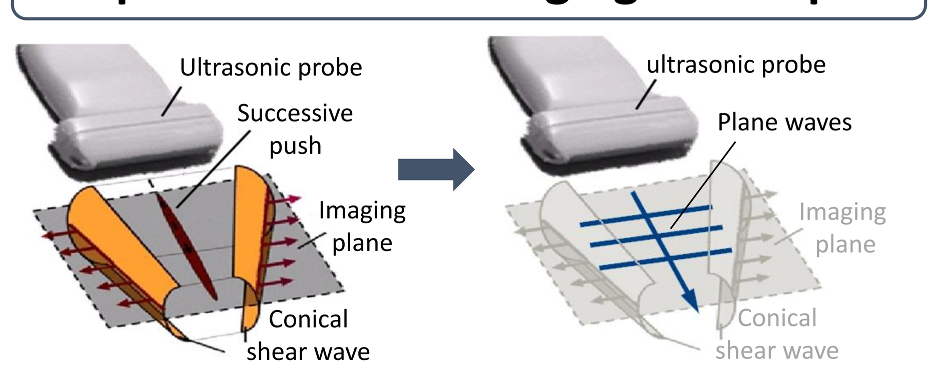
→ **Measuring shear wave speed in TI media under known uniaxial stress.**

Experimental setup for AE experiments

V12 Aixplorer, Supersonic Imagine

3 SL10-2 probes (6 MHz central frequency, 192 elements) were oriented as desired, to carry out measurements in any of the **9 configurations**.

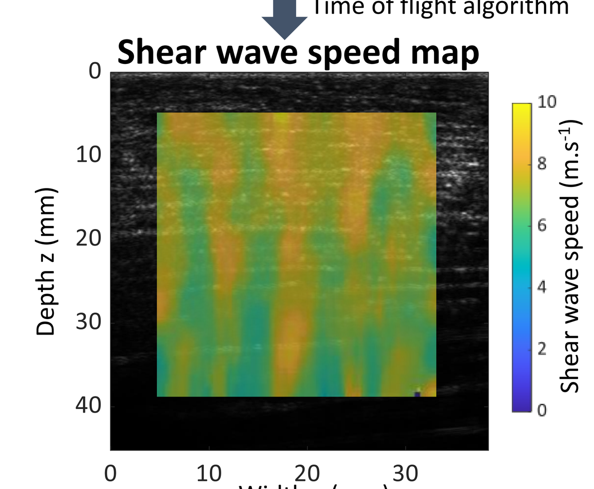
Supersonic Shear Imaging technique



Generation of a supersonic shear wave using the ultrasonic radiation force

Ultrafast imaging of the shear wave propagation (8000 frames.s^{-1}).

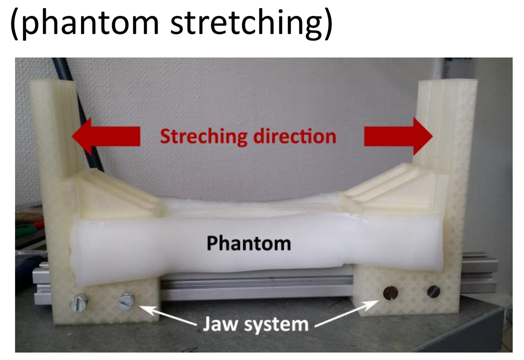
Time of flight algorithm



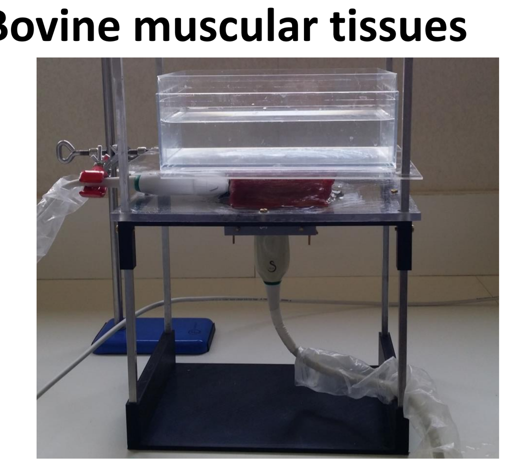
Studied TI media

- **TI PVA phantoms**[4]

  10% PVA – 1% Sigmacell type 20
  3 isotropic freeze-thaw cycles
  3 anisotropic freeze-thaw cycles (phantom stretching)



- **Bovine muscular tissues**



THEORY

→ **Expressing the speed of elastic shear waves in an uniaxially stressed TI quasi-incompressible solid.**

Strain energy[3]:

$$e = \mu_\perp I_2 + \left(\frac{E_\parallel}{2} - \frac{3\mu_\perp}{2}\right)\varepsilon_{33}^2 + 2(\mu_\parallel - \mu_\perp)(\varepsilon_{13}^2 + \varepsilon_{23}^2) + \frac{A}{3} I_3$$
$$+ (G + H + J)\varepsilon_{33}^3 + (H + 2J)\varepsilon_{33}(\varepsilon_{13}^2 + \varepsilon_{23}^2) + J\varepsilon_{33}(\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2)$$

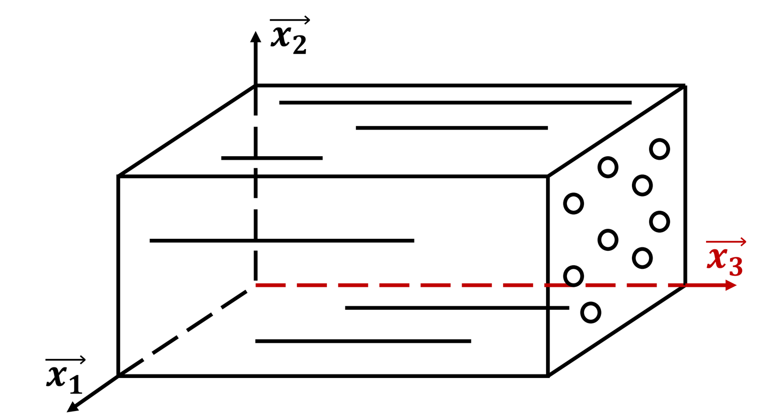
(with ε the Green-Lagrange strain tensor, $I_2 = \mathrm{Tr}\,\varepsilon^2$, $I_3 = \mathrm{Tr}\,\varepsilon^3$)

1st Piola-Kirchhoff stress tensor:
$$P_{ik} = \frac{\partial e}{\partial\left(\frac{\partial u_i}{\partial a_k}\right)}$$

→ Particle motion equation:
$$\rho_0 \ddot{u}_i = \frac{\partial P_{ik}}{\partial a_k}$$

↓ Wave equation

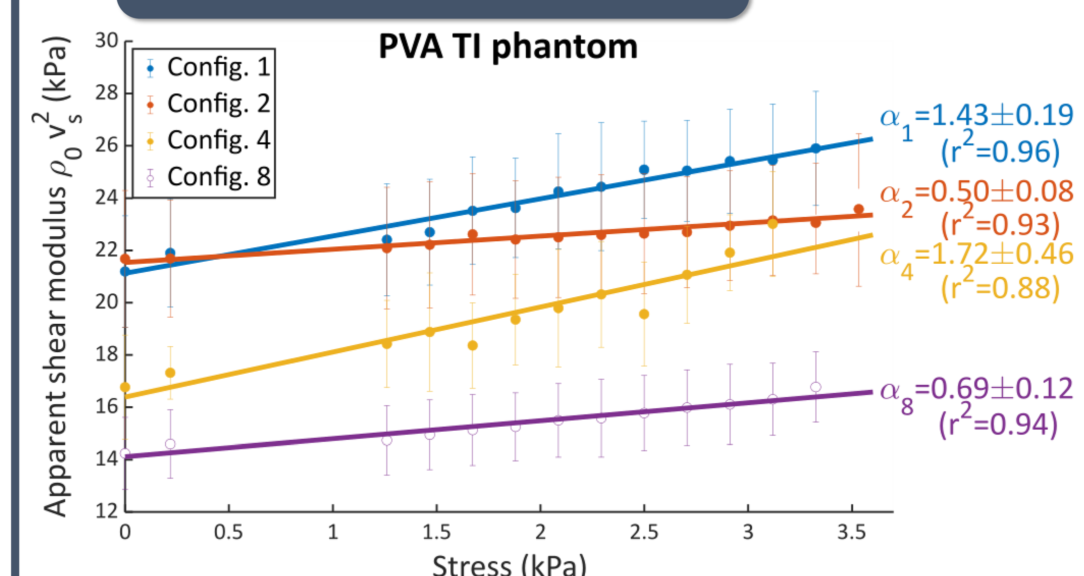
↓ Hooke's law for static strain expression →



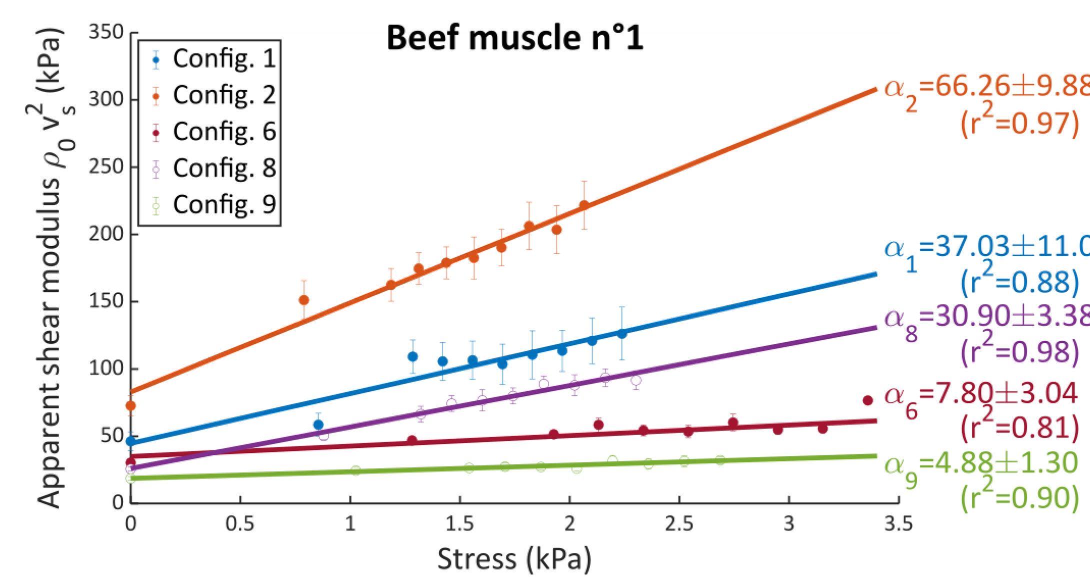
Configuration: direction $\vec{u}$	$\vec{k}$	σ	#	Elastodynamic equations
$\vec{x_2}$	$\vec{x_3}$	$\vec{x_1}$	1	$\rho_0 v_s^2 = \mu_\parallel + \frac{\sigma_{11}}{2E_\parallel}\left(E_\parallel - \mu_\perp + \mu_\parallel + \frac{\mu_\parallel E_\parallel}{\mu_\perp} + \frac{A}{4}\left(1 + \frac{E_\parallel}{\mu_\perp}\right) + \frac{H+2J}{2}\right)$
		$\vec{x_2}$	2	$\rho_0 v_s^2 = \mu_\parallel + \frac{\sigma_{22}}{2E_\parallel}\left(E_\parallel - \mu_\perp + \mu_\parallel - \frac{\mu_\parallel E_\parallel}{\mu_\perp} + \frac{A}{4}\left(1 - \frac{E_\parallel}{\mu_\perp}\right) + \frac{H+2J}{2}\right)$
		$\vec{x_3}$	3	$\rho_0 v_s^2 = \mu_\parallel - \frac{\sigma_{33}}{E_\parallel}\left(E_\parallel - \mu_\perp + \mu_\parallel + \frac{A}{4} + \frac{H+2J}{2}\right)$
$\vec{x_3}$	$\vec{x_2}$	$\vec{x_1}$	4	$\rho_0 v_s^2 = \mu_\parallel + \frac{\sigma_{11}}{2E_\parallel}\left(E_\parallel - \mu_\perp + \mu_\parallel + \frac{\mu_\parallel E_\parallel}{\mu_\perp} + \frac{A}{4}\left(1 + \frac{E_\parallel}{\mu_\perp}\right) + \frac{H+2J}{2}\right)$
		$\vec{x_2}$	5	$\rho_0 v_s^2 = \mu_\parallel + \frac{\sigma_{22}}{2E_\parallel}\left(-E_\parallel - \mu_\perp + \mu_\parallel - \frac{\mu_\parallel E_\parallel}{\mu_\perp} + \frac{A}{4}\left(1 - \frac{E_\parallel}{\mu_\perp}\right) + \frac{H+2J}{2}\right)$
		$\vec{x_3}$	6	$\rho_0 v_s^2 = \mu_\parallel - \frac{\sigma_{33}}{E_\parallel}\left(-\mu_\perp + \mu_\parallel + \frac{A}{4} + \frac{H+2J}{2}\right)$
$\vec{x_2}$	$\vec{x_1}$	$\vec{x_1}$	7	$\rho_0 v_s^2 = \mu_\perp - \frac{\sigma_{11}}{2E_\parallel}\left(E_\parallel + 3\mu_\perp + \frac{A}{2} - J\right)$
		$\vec{x_2}$	8	$\rho_0 v_s^2 = \mu_\perp - \frac{\sigma_{22}}{2E_\parallel}\left(-E_\parallel + 3\mu_\perp + \frac{A}{2} - J\right)$
		$\vec{x_3}$	9	$\rho_0 v_s^2 = \mu_\perp + \frac{\sigma_{33}}{E_\parallel}\left(3\mu_\perp + \frac{A}{2} - J\right)$

Where $\vec{u}$ is the displacement vector, $\vec{a}$ the position in Lagrangian coordinates, $\vec{k}$ the propagation vector, ($\mu_{//}$, $\mu_\perp$, $E_{//}$) the linear elastic coefficients, (A, G, H, J) the third order elastic coefficients, (I_2, I_3) invariant of the strain tensor and ρ_0 the density.

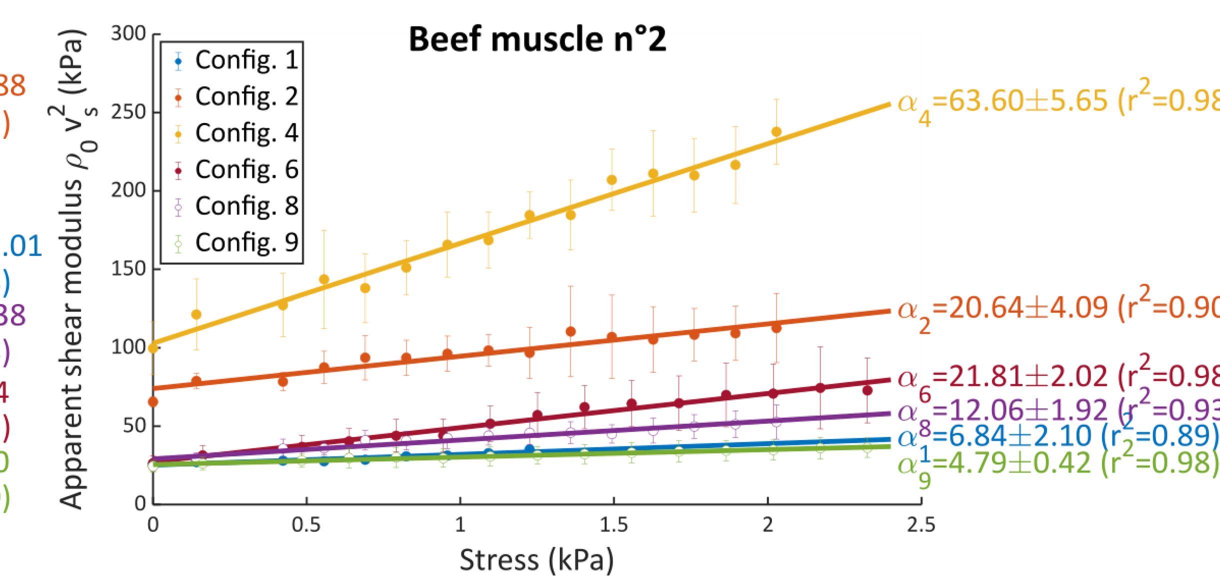
RESULTS



$\mu_\parallel = 21.4 \pm 0.3$ kPa; $\mu_\perp = 14.2 \pm 0.1$ kPa
$A = -33.0 \pm 17.1$ kPa



$\mu_\parallel = 60.0 \pm 19.0$ kPa; $\mu_\perp = 22.1 \pm 5.0$ kPa
$A = -2.8 \pm 2.5$ MPa



$\mu_\parallel = 65.5 \pm 0.4$ kPa; $\mu_\perp = 24.5 \pm 0.1$ kPa
$A = -1.6 \pm 0.6$ MPa

- Linear relations between $\rho_0 v_s^2$ and σ
- Quantification of A

DISCUSSION

The AE theory in TI quasi-incompressible media was derived, leading to the expression of the shear wave speed as a function of stress in 9 specific configurations. Three nonlinear elastic moduli appear in the outcoming equations, along with the 3 linear elastic moduli ($\mu_\parallel$, $\mu_\perp$, $E_\parallel$) of TI media. AE experiments were carried out on TI phantoms and beef muscles and the slopes of the experimental $\rho_0 v_s^2(\sigma)$ curves were used to retrieve the nonlinear elastic moduli A of the studied media.

To fully take advantage of the AE theory and recover H and J, the measurement of $E_\parallel$ is necessary but remains challenging because it requires lateral strain estimation. This lateral strain estimation can be recovered by static elastography technique but remains very sensitive to lateral resolution. Moreover, it is very challenging to control the polarization and propagation direction of shear waves with respect to the fiber axis. Then it is strongly difficult to match experimental position with theoretical configurations. The combination of Backscatter Tensor Imaging (BTI) or Elastic Tensor Imaging (ETI) with AE experiments in TI tissues would help with the exact positioning of the probe and stress with respect to the muscle fibers. This work paves the way to the use of the AE theory to improve muscle characterization for biomechanics, clinics and sport applications.

1/ Bernal M *et al.*, IEEE Trans Ultrason Ferroelectr Freq Control. 2016 ;63(1) :101–109. 2/ Gennisson J-L *et al.*, J Acoust Soc Am. 2008 ;122 :3211–3219. 3/ Johnson GC. J Nondestruct Eval. 1982 ;3(1) :1–8. 4/ Chatelin S *et al.*, Phys Med Biol. IOP Publishing ; 2014 ;59(22) :6923–6940.